Hypothesis Testing – One Sample Tests

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Test of the Parameter μ of a Normal Distribution

Let $X_1, X_2, \dots, X_n$ be a random sample from a normal distribution $N(\mu, \sigma^2)$. A statistic

$$T = \frac{\overline{X} - \mu}{S}\sqrt{n}$$

has a Student distribution with $\nu = n - 1$ degrees of freedom. We use this statistic for testing of the parameter μ.

Test of the Parameter μ of a Normal Distribution

Let $x_1, x_2, \ldots, x_n$ be values of a random sample (measured data), $\overline{x}$ denotes an arithmetic mean and s a sample standard deviation. We test the hypothesis that the parameter μ is equal to a constant μ_0:

$$H : \mu = \mu_0,$$

the test statistic

$$t = \frac{\overline{x} - \mu_0}{s}\sqrt{n},$$

has under the null hypothesis H a Student t-distribution with $\nu = n - 1$ degrees of freedom.

Test of the Parameter μ of a Normal Distribution

According to the alternative hypothesis we construct following regions of rejection:

alternative hypothesis	rejection region
$A: \mu > \mu_0$	$W_\alpha = \{t, t \geq t_{1-\alpha}(\nu)\}$
$A: \mu < \mu_0$	$W_\alpha = \{t, t \leq -t_{1-\alpha}(\nu)\}$
$A: \mu \neq \mu_0$	$W_\alpha = \left\{t, \lvert t \rvert \geq t_{1-\frac{\alpha}{2}}(\nu)\right\}$

where $t_{1-\alpha}(\nu)$, $t_{1-\frac{\alpha}{2}}(\nu)$ are quantiles of the Student distribution.

Test of the Parameter σ^2 of a Normal distribution

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution $N(\mu, \sigma^2)$. A statistic

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has a Pearson distribution with $\nu = n - 1$ degrees of freedom. We use this statistic for testing of the parameter σ^2.

Test of the Parameter σ^2 of a Normal distribution

Let $x_1, x_2, \dots, x_n$ be values of a random sample (measured data), s^2 denotes a sample variance.
We test the hypothesis that the parameter σ^2 is equal to a constant σ_0^2:

$$H : \sigma^2 = \sigma_0^2,$$

the test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

has under the null hypothesis H a Pearson χ^2-distribution with $\nu = n - 1$ degrees of freedom.

Test of the Parameter σ^2 of a Normal distribution

According to an alternative hypothesis we construct following regions of rejection:

alternative hypothesis	rejection region
$A: \sigma^2 > \sigma_0^2$	$W_\alpha = \{\chi^2, \chi^2 \geq \chi^2_{1-\alpha}(\nu)\}$
$A: \sigma^2 < \sigma_0^2$	$W_\alpha = \{\chi^2, \chi^2 \leq \chi^2_{\alpha}(\nu)\}$
$A: \sigma^2 \neq \sigma_0^2$	$W_\alpha = \left\{\chi^2, \chi^2 \leq \chi^2_{\frac{\alpha}{2}}(\nu) \text{ or } \chi^2 \geq \chi^2_{1-\frac{\alpha}{2}}(\nu)\right\}$

where $\chi^2_{1-\alpha}(\nu)$, $\chi^2_{1-\frac{\alpha}{2}}(\nu)$ are quantiles of the Pearson χ^2-distribution.

Large Sample Test on Mean

Let $X_1, X_2, \ldots, X_n$ be a random sample from any distribution with the mean μ. A statistic

$$U = \frac{\overline{X} - \mu}{S}\sqrt{n}$$

has for large n approximately a normal distribution $N(0, 1)$ – see the central limit theorems. We use this statistic for testing of the parameter μ.

Large Sample Test on Mean

Let $x_1, x_2, \ldots, x_n$ be values of a random sample (measured data), $\overline{x}$ denotes an arithmetic mean and s a sample standard deviation. We test the hypothesis that the parameter μ is equal to a constant μ_0:

$$H : \mu = \mu_0,$$

the test statistic

$$u = \frac{\overline{x} - \mu_0}{s}\sqrt{n},$$

has under the null hypothesis H asymptotically a normal distribution $N(0,1)$.

Large Sample Test on Mean

According to an alternative hypothesis we construct following regions of rejection:

alternative hypothesis	rejection region
$A: \mu > \mu_0$	$W_\alpha = \{u, u \geq u_{1-\alpha}\}$
$A: \mu < \mu_0$	$W_\alpha = \{u, u \leq -u_{1-\alpha}\}$
$A: \mu \neq \mu_0$	$W_\alpha = \left\{u, \lvert u \rvert \geq u_{1-\frac{\alpha}{2}}\right\}$

where $u_{1-\alpha}$, $u_{1-\frac{\alpha}{2}}$ are quantiles of $N(0,1)$.

Test on a Population Proportion

Suppose that a random sample of size n has been taken from a large (possibly infinite) population and that m observations in this sample belong to a class of interest. Then $p = \frac{m}{n}$ is a point estimator of the proportion of the population π that belongs to this class. A random variable

$$U = \frac{\hat{\pi} - \pi}{\sqrt{\pi(1-\pi)/n}}$$

has for $n \to \infty$ approximately a normal distribution $N(0,1)$ – see central limit theorems. We use this statistic for testing of the population proportion.

Test on a Population Proportion

Let $\hat{\pi} = \frac{m}{n}$ be a point estimator of population proportion.
We test the hypothesis that the parameter π is equal to a constant π_0:

$$H : \pi = \pi_0,$$

a test statistic

$$u = \frac{\hat{\pi} - \pi_0}{\sqrt{\pi_0(1 - \pi_0)/n}}$$

has under the null hypothesis H asymptotically a normal distribution $N(0, 1)$.

Test on a Population Proportion

According to an alternative hypothesis we construct following regions of rejection:

alternative hypothesis	rejection region
$A: \pi > \pi_0$	$W_\alpha = \{u, u \geq u_{1-\alpha}\}$
$A: \pi < \pi_0$	$W_\alpha = \{u, u \leq -u_{1-\alpha}\}$
$A: \pi \neq \pi_0$	$W_\alpha = \left\{u, \lvert u \rvert \geq u_{1-\frac{\alpha}{2}}\right\}$

where $u_{1-\alpha}$, $u_{1-\frac{\alpha}{2}}$ are quantiles of $N(0,1)$.