1 DISCRETE RANDOM VARIABLES AND THEIR PROBABILITY DISTRIBUTIONS

1.1 Poisson Probability Distribution

1.1.1 Exercises 1.1

1. An automatic machine is randomly out of order during a shift. It is well known by an experience that the machine breaks down an average of two times a shift. What is the probability that the machine is trouble-free for 24 hours (i.e. 3 shifts)?
2. There are, on an average, 20 visitors during one hour in the in an inquiry office. Compute the probability that nobody comes during next 15 minutes. Consider the number of visitors follows the Poisson distribution.
3. There are, on an average, 360 telephone exchange telephone calls during 8 hours. What is the probability that during next 10 minutes will be
   a) 4 calls,
   b) most highly 4 calls?
4. Seeds of certain plant are polluted with a modicum of weed. It is well known by an experience that 4 weeds grow in 1 m^2.
   a) Describe a random variable „the number of weeds in 1 m^2“ by means of the probability function and the distribution function (including graphs).
   b) Compute the probability that in a randomly chosen m^2 are not weeds, are most highly 3 weeds, are at least 3 weeds.
   c) Compute the mean, the variance, the standard deviation, the mode, the skewness and the kurtosis of the monitored random variable.
5. There are, on an average, 40 mistakes per 100 pages in a book.
   a) What is the probability that in randomly chosen 20 pages are at lest 5 mistakes, most highly 10 mistakes, between 5 and 10 mistakes?
   b) Determine the mean and a the likeliest number of mistakes in 20 pages.

Solution.
1. 0.00248;
2. 0.00674;
3. a) 0.073; b) 0.132;
4. a) $p(x)$: $\frac{4^x}{x!}e^{-4}$ for $x = 0, 1, 2, \ldots$; 0 otherwise; b) 0.018; 0.433; 0.567; c) 4; 4; 2; 3 a 4; 0.5; 0.25;
5. a) 0.809; 0.816; 0.716; b) 8; 7 a 8.

1.2 Alternative distribution

1.3 Binomial distribution

1.3.1 Exercises 1.2 a 1.3

1. A player throws a fair die three times. The random variable X represents the number of outcomes 6.
   a) Describe the random variable by means of the probability function and the distribution function (including graphs).
   b) What is the probability that the outcome 6 occurs either once or twice, at lest once, most highly twice.?
   c) Compute the mean, the mode, the variance, the standard deviation, the skewness and the kurtosis of the monitored random variable.
2. We plant 10 seeds of given vegetable. Consider the successfulness of seeds is 80 %.
   a) What is the likeliest number of sound seeds? What is the probability that we grow this number of seeds?
   b) Compute the probability that we grow at lest 5 plants, most highly 9 plants, between 4 and 8 plants?
3. A worker is responsible for 800 spindles which are intended for winding wool. The probability of rupture is for each spindle 0.005. What is the probability that at least two spindles are ruptured? Verify if the Poisson distribution could be used as an approximation to the probability function of described random variable? In case of the approximation, compute the task by the help of Poisson distribution.
4. An air company runs an aeroplane for 120 passengers. It is common practice that 3 % of passengers do not appear, on an average, on board the plane despite all seats are booked. The company takes decision to allow booking for 122 passengers. What is the probability that the company can not satisfy passengers requests?
5. A student is about to write a test, but he has not studied. He chooses answers (possibilities yes–no) randomly. The test consists of 20 questions and for passing the exam is required 15 correct answers. What is the probability that he will pass the test?

Solution.
1. a) $p(x)$: $\binom{3}{x}\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{3-x}$ for $x = 0, 1, 2, 3$; 0 otherwise; b) 0.417; 0.421; 0.995; c) 0.5; 0; 0.417; 0.645; 1.033; 0.4;
2. a) 8; 0.302; b) 0.994; 0.893; 0.623;
3. 0.76263; $Po(4)$; 0.76190;
4. 0.116;
5. 0.021.

1.4 Hypergeometric distribution

1.4.1 Exercises 1.4

1. There are 49 balls with different numbers in the wheel of fortune, draw (without replacement) 6 of them. The player marks 6 chosen numbers on a ticket. The random variable X represents „the number of correct chosen balls“.
   a) Describe the random variable by means of the probability function and the distribution function (including graphs).
   b) Compute the probability that the player guesses correctly all 6 numbers.
   c) What is the probability that the player does not win (i.e. guesses correctly most highly 2 numbers)?
   d) Compute the mean and the mode of the monitored random variable.
2. A certain kind of components is delivered in a packing where is exactly 200 pieces of components. Each packing is checked so that 5 components are randomly chosen and the packing is accepted if all components are correct. Consider the packing contains 10 broken components. The random variable X represents „the number of broken chosen components“.
   a) Describe the random variable X by means of the probability function and the distribution function (including graphs).
   b) What is the probability that the packing is accepted?
   c) Compute the mean, the variance, the standard deviation and the mode of the monitored random variable.
   d) Verify conditions of an approximation to the different kind of distribution.
3. In a delivery of 80 products are 8 defective products. Choose randomly 5 products for next completion. What is the probability that among chosen products is most highly one defective product?
4. There is 30 students in the group and 6 of them are excellent. Choose 20 students because of a specific course. What is the probability that in the course
   a) are all excellent students,
   b) are at least 3 excellent students?
5. There are 20 red and 30 blue balls in the urn. Choose randomly 8 balls. What is the probability that among chosen balls are 4 blue balls given that we choose
   a) without replacement,
   b) with replacement?

Solution.
1. a) $p(x)$: $\binom{6}{x}\binom{43}{6-x}/\binom{49}{6}$ for $x = 0, 1, \ldots, 6$; otherwise 0; b) $7.151 \cdot 10^{-8}$; c) 0.981; d) 0.735; 0;
2. a) $p(x)$: $\binom{10}{x}\binom{190}{5-x}/\binom{200}{5}$ for $x = 0, 1, \ldots, 5$; otherwise 0; b) 0.772; c) 0.25; 0.233; 0.482; 0; d) $B(5; 0.05)$;

3. 0.924;
4. a) 0.065; b) 0.924;
5. a) 0.247; b) 0.232.