Selected Methods of Economic Time Series Description

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Abstract: The article is focused on selected quantitative methods which can be used for description of economic time series. Vector autoregressive models and cointegration analysis play an important role in description of economic events. Multidimensional non-stationary process is called cointegrated if there is a linear combination of its one-dimensional components, which is stationary or trend-stationary. Economic time series are predominantly non-stationary, nevertheless, one can find linear links which keep that whole system in so-called long-term equilibrium. The Granger causality test is employed to analyze causal relations between time series. Next useful tools for analysis of economic time series are methods of change point detection. Authors compare standard statistical methods of change point detection with the method based on the sparse parameter estimation. All mentioned methods are applied to real economic data sets.

Keywords: cointegration, Granger causality, change point

1. Introduction

The authors, such as Benoit (1973) and Kollias et al. (2004), refer to the existence of links between military expenditure and economic growth in selected groups of countries and various time periods. According to Kollias et al. (2004), it is not possible to generalize this link and it is therefore necessary to analyse this theoretical relation between economic variables described in Kollias et al. (2004) and Dunne et al. (2005) while taking into consideration that military expenditure can have both positive effect as well as adverse effect on the economy of the given country. Military expenditure proper as a part of government spending can, according to Kollias et al. (2004), influence the economy in various possible ways. Stimulating economic growth by means of the multiplication effect of government spending in periods when the economy was under the so called potential product, was one of the instruments of Keynesian Economics. The negative effect of military expenditure on economic growth is referred to in specialist studies (see Dunne et al. (2005)) as the crowding out effect where military expenditure results in crowding out part of capital expenditure due to increased interest rate. The "crowded out" investments fail to contribute to GDP and therefore to the economic growth of the given economy. In theory it is possible to distinguish 4 types of link between military expenditure and economic growth:

a) mutual link between anticipated variables,

b) link showing influence of military expenditure on economic growth,

c) link showing influence of economic growth on the level of military expenditure,

d) non-existence of any link between anticipated variables.

The purpose of this paper is to analyse the long-term temporal series of military expenditure and economic growth and to prove the existence of the above theoretical links on realistic data by means of structural analysis (Granger causality) in VAR models and in VECM models for cointegrated time series (see Lüthepohl (2007)). To analyse the link between military expenditure and economic growth, temporal series of military expenditure expressed as a percentage of GDP from the SIPRI database and economic growth (the growth rate in per cent) from the OECD database have been selected. To assess the existence of a link between the two variables, German economy has been selected as a suitable economy characterized by periods of economic growth as well as downturn and stable level of military expenditure in percentage of GDP. The selected temporal series describe the period from 1953 to 2009. Selected methods of change point detection such as statistical methods, a basis pursuit approach and ℓ_1-trend filtering are applied to the time series of military expenditure in Greece and Germany.

2. Cointegration and Granger causality

In this part of the contribution we describe selected methods of time series analysis which will be used later.

Definition 1. Let $\{\boldsymbol{\epsilon}_t\}$ be a set of independent identically distributed random variables with zero mean and variance matrix $\mathbf{\Omega}$. A stochastic process $\boldsymbol{Y}_t$ which satisfies that $\boldsymbol{Y}_t - E\boldsymbol{Y}_t =$ $= \sum_{i=1}^{\infty} \mathbf{C}_i \boldsymbol{\epsilon}_{t-i}$ is called $I(0)$ process if $\mathbf{C} = \sum_{i=0}^{\infty} \mathbf{C}_i \neq 0$.

Definition 2. A stochastic process $\{\boldsymbol{Y}_t\}$ is called *integrated of order* d, $I(d)$, $d = 1, 2, \ldots,$ if $\Delta^d(\boldsymbol{Y}_t - E\boldsymbol{Y}_t)$ is $I(0)$ process.

Let in the following ϵ_t be a sequence of independent normally distributed n-dimensional random variables $\epsilon_t \sim N_n(\mathbf{0}, \mathbf{\Omega})$.

Definition 3. A stochastic process $\{\boldsymbol{Y}_t\}$ is called n-dimensional autoregressive process VAR(p), if

$$\boldsymbol{Y}_t = \mathbf{\Phi}_1 \boldsymbol{Y}_{t-1} + \mathbf{\Phi}_2 \boldsymbol{Y}_{t-2} + \cdots + \mathbf{\Phi}_p \boldsymbol{Y}_{t-p} + \mathbf{\Lambda} \mathbf{D}_t + \boldsymbol{\epsilon}_t, t = 1, 2, \ldots, T \tag{1}$$

for fixed values of $\boldsymbol{Y}_{-p+1}, \ldots, \boldsymbol{Y}_0$,where $\mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_p$ are matrices of coefficients $(n \times n)$, $\mathbf{\Lambda}$ is an $(n \times s)$ matrix of coefficient of deterministic term $\mathbf{D}_t$ $(s \times 1)$, which can contain a constant, a linear term, seasonal dummies, intervention dummies or other regressors that we consider non-stochastic.

The process defined by the equation (1) can be written in *error correction* form (VECM)

$$\Delta \boldsymbol{Y}_t = \mathbf{\Pi} \boldsymbol{Y}_{t-1} + \sum_{i=1}^{p-1} \mathbf{\Gamma}_i \Delta \boldsymbol{Y}_{t-i} + \mathbf{\Lambda} \mathbf{D}_t + \boldsymbol{\epsilon}_t,\ t = 1, \ldots, T, \tag{2}$$

where $\mathbf{\Pi} = \sum_{i=1}^{p} \mathbf{\Phi}_i - \mathbf{I}$, $\mathbf{\Gamma}_i = -\sum_{j=i+1}^{p} \mathbf{\Phi}_j$. This error correction form of VAR process is used in the analysis of cointegration.

The basic idea of cointegration can be shown on 2 one-dimensional processes of order $I(1)$. We say that the processes X_t a Y_t are cointegrated if exists any linear combination $aX_t + bY_t$ which is stationary.

Definition 4. Let $\boldsymbol{Y}_t$ be n-dimensional process integrated of order 1. We call this process *cointegrated with a cointegrating vector* $\boldsymbol{\beta}$ $(\boldsymbol{\beta} \in R^n, \boldsymbol{\beta} \neq 0)$ if $\boldsymbol{\beta}' \boldsymbol{Y}_t$ can be made stationary by a suitable choice of its initial distribution.

The basic test of cointegration based on the maximum likelihood estimation (so called MAX and TRACE tests) are described in Johansen (1995) or Lüthepohl (2007).

Granger causality

The idea of *Granger causality* can be expressed as follows: If a variable Y affects a variable Z, the former should help improving the prediction of the later variable. To formalize this idea, suppose that Ω_t is the information set containing all relevant information available up to and including period t. Let $Z_t(h|\omega_t)$ be the optimal (minimum MSE) h-step predictor of the process Z_t at origin t, based on the information in Ω_t. The corresponding forecast MSE will be denoted $\Sigma_Z(h|\Omega_t)$. The process Y_t is said to *cause* Z_t *in the Granger sense* if

$$\Sigma_Z(h|\Omega_t) < \Sigma_Z(h|\Omega_t - \{Y_s|s \leq t\}) \text{ for at least one } h = 1, 2, \ldots.$$

The expression $\Omega_t - \{Y_s|s \leq t\}$ is a set containing all relevant information except for the information in the past and the present of the process Y_t

Assume the two-dimensional stable VAR process

$$\begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \Phi_{11}^1 & \Phi_{12}^1 \\ \Phi_{21}^1 & \Phi_{22}^1 \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + \cdots + \begin{bmatrix} \Phi_{11}^p & \Phi_{12}^p \\ \Phi_{21}^p & \Phi_{22}^p \end{bmatrix} \begin{bmatrix} Y_{t-p} \\ Z_{t-p} \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}.$$

In this model it can be seen (see Lüthepohl (2007)) that Y_t does not Granger cause Z_t if and only if $\Phi_{21}^i = 0$ for $i = 1, \ldots, p$; analogously Z_t does not Granger cause Y_t if and only if $\Phi_{12}^i = 0$, $i = 1, \ldots, p$. If one wants to test Granger causality, the usual F-statistic for a regression model can be used (see Lüthepohl (2007)). It is easy to derive the corresponding restrictions for the error correction form (VECM)

$$\begin{bmatrix} \Delta Y_t \\ \Delta Z_t \end{bmatrix} = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + \sum_{i=1}^{p-1} \begin{bmatrix} \Gamma_{11}^i & \Gamma_{12}^i \\ \Gamma_{21}^i & \Gamma_{22}^p \end{bmatrix} \begin{bmatrix} \Delta Y_{t-i} \\ \Delta Z_{t-i} \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}.$$

Then Y_t does not Granger cause Z_t if and only if $\Pi_{21} = 0$ and $\Gamma_{21}^i = 0$, $i = 1, \ldots, p-1$. In case of cointegrated processes, testing these restrictions is not as straightforward as for stationary processes (see Lüthepohl (2007)).

3. Change point detection

Let us assume the model with p change points

$$Y_t = \begin{cases} \mu + \epsilon_t & \text{for} \quad t = 1, 2, \ldots, c_1 \\ \mu + \delta_1 + \epsilon_t & \quad\quad t = c_1 + 1, \ldots, c_2, \\ \cdots & \quad\quad \cdots \\ \mu + \delta_p + \epsilon_t & \quad\quad t = c_p + 1, \ldots, T, \end{cases} \tag{3}$$

where $\mu, \delta_1, \ldots, \delta_p \neq 0, t_0 \leq c_1 < \cdots < c_p < T - t_0$ are unknown parameters and ϵ_t are independent identically distributed random variables with zero mean and variance σ^2.

Statistical methods

Consider the model (3) with only one change point c. Assuming σ^2 given, the unknown parameters c, μ and δ may be estimated by the least-squares method. The least-squares estimators $\hat{c}$, $\hat{\mu}$ and $\hat{\delta}$ of the parameters c, μ and δ are defined as solutions of the minimization problem

$$\min \left\{ \sum_{t=1}^{k} (Y_t - \mu)^2 + \sum_{t=k+1}^{T} (Y_t - \mu - \delta)^2; k \in \{1, \ldots, T-1\}, \mu \in \mathbb{R}, \delta \in \mathbb{R} \right\}.$$

In other words, the unknown parameters are estimated in such a way that the sum of squares of residuals is minimal. The estimates of the parameters μ and δ are (see Antoch et al. (2000), Csörgö and Horváth (1997))

$$\hat{\mu} = \overline{Y}_{\hat{c}} \quad \text{and} \quad \hat{\delta} = \overline{Y}^{0}_{\hat{c}} - \overline{Y}_{\hat{c}},$$

where $\hat{c}$ is a solution of the maximization problem

$$\hat{c} = \arg\max \left\{ \sqrt{\frac{T}{k(T-k)}} \cdot |S_k| \, ; k \in \{1, \ldots, T-1\} \right\}, \tag{4}$$

where we denote $S_k = \sum_{t=1}^{k}(Y_t - \overline{Y}_T)$, $\overline{Y}_T = \frac{1}{T}\sum_{t=1}^{T} Y_t$, $\overline{Y}_{\hat{c}} = \frac{1}{\hat{c}}\sum_{t=1}^{\hat{c}} Y_t$ and $\overline{Y}^{0}_{\hat{c}} = \frac{1}{T-\hat{c}}\sum_{t=\hat{c}+1}^{T} Y_t$. In the case of multiple change points, we can use the described statistical method of one change point detection via the following procedure. At first, find $\hat{c}^{(1)}$ by solving (4). Secondly, divide observations into two groups $Y_1, \ldots, Y_{\hat{c}^{(1)}}$ and $Y_{\hat{c}^{(1)}+1}, \ldots Y_T$ and find the estimator in each group. The whole procedure is repeated until a "constant mean is obtained". This procedure is called "binary segmentation".

Basis pursuit approach

We briefly describe the method based on basis pursuit algorithm (BPA) for the detection of the change point in the sample path $\{y_t\}$ in one-dimensional stochastic process $\{Y_t\}$. We assume a deterministic functional model on a bounded interval $\mathcal{I}$ described by the dictionary $G = \{G_j\}_{j \in J}$ with atoms $G_j \in L^2(\mathcal{I})$ and with additive white noise e on a suitable finite discrete mesh $\mathcal{T} \subset \mathcal{I}$:

$$Y_t = x_t + e_t, \; t \in \mathcal{T},$$

where $x \in \operatorname{sp}(\{G_j\}_{j \in J})$, $\{e_t\}_{t \in \mathcal{T}} \sim WN(0, \sigma^2)$, $\sigma > 0$, and J is a big finite indexing set. Smoothed function $\hat{x} = \sum_{j \in J} \hat{\xi}_j G_j =: \mathbf{G}\hat{\xi}$ minimizes on $\mathcal{T}$ ℓ_1-penalized optimality measure $\frac{1}{2}\|\mathbf{y} - \mathbf{G}\xi\|^2$ as follows:

$$\hat{\xi} = \arg\min_{\xi \in \ell^2(J)} \frac{1}{2}\|\mathbf{y} - \mathbf{G}\xi\|^2 + \lambda\|\xi\|_1, \; \|\xi\|_1 := \sum_{j \in J} \|G_j\|_2 \xi_j,$$

where $\lambda = \sigma\sqrt{2 \ln(\operatorname{card} J)}$ is a smoothing parameter. Such approaches are also known as basis pursuit denoising (BPDN). Solution of this minimization problem with λ close to zero may not be sparse enough: we are searching small $F \subset J$ such that $\hat{x} \approx \sum_{j \in F} \hat{\xi}_j G_j$ is a good approximation. The procedure of BPDN is described in Neubauer and Veselý (2011).

We build our dictionary from heaviside-shaped atoms on $L^2(\mathbb{R})$ derived from a fixed 'mother function' via shifting and scaling following the analogy with the construction of wavelet bases. We construct an oversized shift-scale dictionary $G = \{G_{a,b}\}_{a \in \mathcal{A}, b \in \mathcal{B}}$ derived from the 'mother function' by varying the shift parameter a and the scale (width) parameter b between values from big finite sets $\mathcal{A} \subset \mathbb{R}$ and $\mathcal{B} \subset \mathbb{R}^+$, respectively ($J = \mathcal{A} \times \mathcal{B}$), on a bounded interval $\mathcal{I} \subset \mathbb{R}$ spanning the space $H = \operatorname{sp}(\{G_{a,b}\})_{a \in \mathcal{A}, b \in \mathcal{B}}$, where

$$G_{a,b}(t) = \begin{cases} 1 & \text{for} \quad t - a > b/2, \\ 2(t-a)/b & \qquad\quad |t-a| \le b/2, b > 0, \\ 0 & \qquad\quad t = a, b = 0, \\ -1 & \qquad\quad \text{otherwise.} \end{cases}$$

Some examples of Heaviside functions are displayed in figure 1. The shift parameters of the significant atoms (the atoms contained in solution by BPDN) indicates possible change points in the sample path of the process.

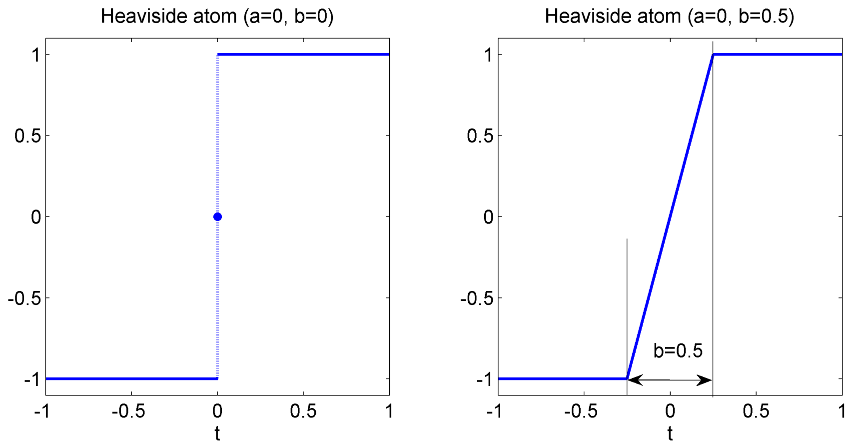

Figure 1: Heaviside atoms with parameters $a = 0, b = 0$ and $a = 0, b = 0.5$

ℓ_1-trend filtering

In Kim et al. (2009) the authors propose a variation on Hodrick-Prescott filtering, which they call ℓ_1-trend filtering. The trend is estimated as the minimizer of the objective function

$$\frac{1}{2}\sum_{t=1}^{T}(y_t - x_t)^2 + \chi\sum_{t=2}^{T-1}|x_{t-1} - 2x_t + x_{t+1}|, \tag{5}$$

which can be written in the matrix form as

$$\frac{1}{2}\|\mathbf{y} - \mathbf{x}\|_2^2 + \chi\|\mathbf{D}\mathbf{x}\|_1,$$

where $\chi \geq 0$ is a smoothing parameter, $\mathbf{x} = (x_1, \ldots, x_T)' \in \mathbb{R}^T$, $\mathbf{y} = = (y_1, \ldots, y_n)' \in \mathbb{R}^T$ and $\mathbf{D} \in \mathbb{R}^{(T-2)\times T}$ is the matrix

$$\mathbf{D} = \begin{pmatrix} 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & 1 & -2 & 1 & \\ & & & & 1 & -2 & 1 \end{pmatrix}.$$

The ℓ_1 trend estimate is piecewise linear in t. The points where the slope of the estimated trend is changed can be interpreted as abrupt changes (change points) in the process. The argument appearing in the second term of (5), $x_{t-1} - 2x_t + x_{t+1}$, is the second difference of $\{x_t\}$. It is zero when and only when three points x_{t-1}, $2x_t$ and x_{t+1} are on the line. This method can be used for change point detection in the linear regression.

4. Real data example

The first part of this paragraph deals with analysis of the time series of GDP (index) describing economic growth and military spending in Germany from 1953 to 2009. In the second part we apply selected methods of change point detection to the time series of military expenditure in Greece (from 1949 to 2009) and Germany.

The time series of economic growth (GDP index) and military spending (percentige of GDP) in Germany are displayed in figure 2. We would like to answer the question whether there is any causality between those two macroeconomic indicators. For this purpose we employ the Granger causality test. At first we test the stationarity of these time series using unit roots tests (the Augmented Dickey-Fuller test and the KPSS test). The null hypothesis of the augmented Dickey-Fuller (ADF) test is that the generated process is non-stationary $I(1)$ process, the null hypothesis of the KPSS test is opposite to that in the ADF: under the null hypothesis, the the process is stationary; the alternative is that is $I(1)$. The results of the mentioned test are summarized in tables. 1 and 2.

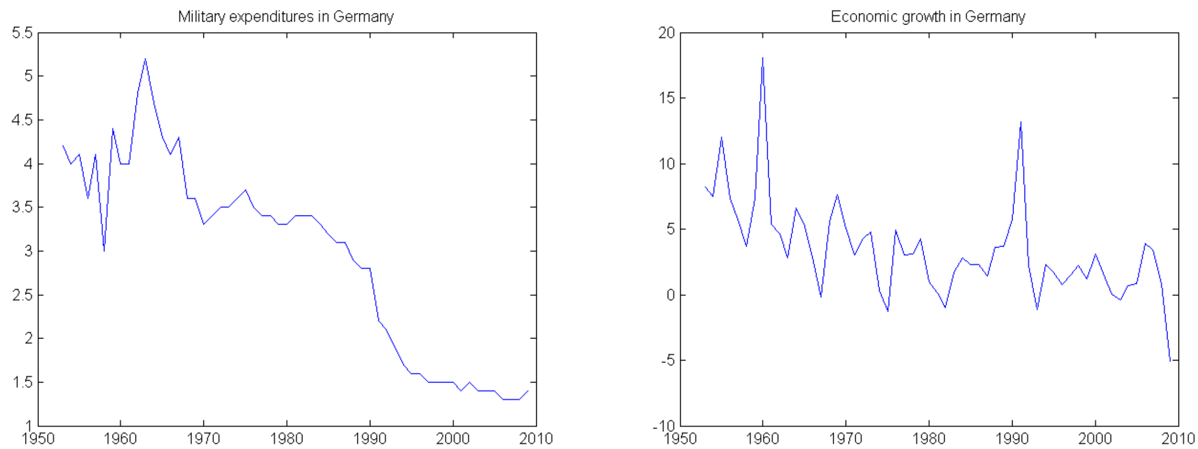

Figure 2: The time series of military expenditure and economic growth in Germany

	lag	test statistic	p-value
military expenditures	2	-0.4297	0.9019
first difference	1	-5.7742	$4.105 \cdot 10^{-7}$
economic growth	1	-3.7538	0.0034
first difference	4	-4.9944	$2.096 \cdot 10^{-5}$

Table 1: The ADF unit root tests of military expenditure and economic growth in Germany

	test statistic	10%	5%	1%
military expenditure	0.9531	0.351	0.468	0.726
first difference	0.0742			
economic growth	0.7452			
first difference	0.0798			

Table 2: The KPSS unit root tests of military expenditure and economic growth in Germany

Based on these results, these two time series can be considered as the non-stationary ($I(1)$ processes). In the next step we perform the cointegration analysis of the two-dimensional time series consisting of the time series of military expenditure and economic growth. This time series can be described as the VAR(2) model with a constant. Table 3 contains the outputs of the cointegration analysis leading to the conclusion that the two dimensional process is cointegrated with one cointegrating vector. (The hypothesis that there is no cointegrating vector ($r = 0$) is rejected by both tests, the hypothesis $r = 0$ in not rejected.)

r	eigenvalue	TRACE	p-value	MAX	p-value
0	0.28499	18.504	0.0156	18.451	0.0087
1	0.00098	0.0537	0.8167	0.0537	0.8167

Table 3: The tests of cointegration for military expenditure and economic growth in Germany

The Granger-causality test results are shown in table 4. The test were calculated for the VAR and VECM model. We can say that at the significance level 0,05 we reject all null hypothesis meaning that economic growth Granger-causes military expenditure, and vice versa. At then significance level 0.01 we come to the conclusion that only military expenditures Granger-causes economic growth (the VAR model).

causality tests	VAR		VECM	
	test statistic	p-value	test statistic	p-value
H_0: economic growth do not Granger-cause military expenditures	4.0099	0.0211	4.2425	0.0172
H_0: military exp. do not Granger-cause economic growth	5.6302	0.0048	3.3397	0.0397

Table 4: The Granger causality tests of economical growth and military expenditure in Germany

The second part ot this paragraph is focused on the problem of change point detection. Firstly we analyze the time series of military expenditure in Greece (1949–2009). We apply the method based on the basis pursuit algorithm (BPDN) and one statistical method of multiple change point detection (based on the binary segmentation). The results obtained are presented in table 5. One can see that we get the similar outputs except for the first and third (sorted) estimates, where the BPDN method gives the change points 1953, 1968 and the binary segmentation estimates are 1957 and 1967 (see figure 3).

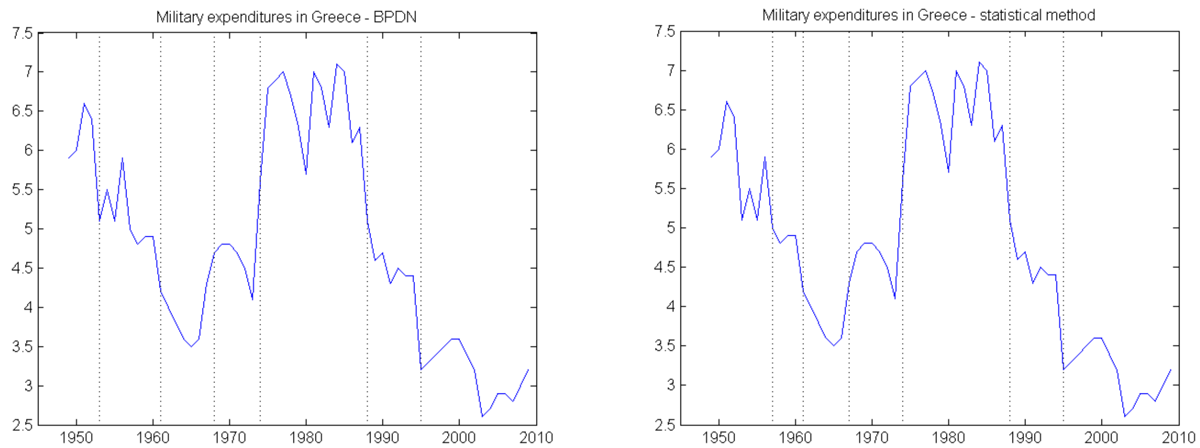

Figure 3: Results of change point detection in the time series of military expenditure in Greece

method	change points
BPDN	1988, 1995, 1974, 1953, 1961, 1968
BPDN (sorted)	1953, 1961, 1968, 1974, 1988, 1995
binary segmentation	1995, 1974, 1988, 1957, 1961, 1967
binary segmentation (sorted)	1957, 1961, 1967, 1974, 1988, 1995

Table 5: The results of change point detection in the time series of military expenditures in Greece

The method of ℓ_1-trend filtering we apply to the time series of military expenditures in Germany. The result is displayed in figure 4. In this case the smoothing parameter was set to $\chi = 1$. According to this method we can estimate changes in the linear regression in the years: 1958, 1963, 1970, 1984, 1987 and 1995. It should be noted that the result is dependent on the value of the smoothing parameter χ.

Figure 4: ℓ_1- trend filtering of military expenditure in Germany

5. Conclusion

The aim of this paper was to investigate the existence of links between military expenditure and economic growth in the case of Germany. To analyse mutual links between the above variables, the Granger causality tests have been used with the intention of proving the existence of a link between military expenditure and economic growth and especially the direction of this link proving the link between the economic situation in the given country and military expenditure in the period when Germany is benefiting from its membership in NATO and is not exposed to any imminent danger to the safety of the state. Economic factors can, therefore, be considered decisive factors determining the level of German military expenditure. In accordance with the result of the Granger causality tests we can say that at the significance level 0.05 we reject all null hypothesis meaning that economic growth Granger-causes military expenditures, and vice versa. The economic time series indicates that within the analysis of the economic growth variable, it is possible to consider the efficiency of German economy during the post-war reconstruction. The particular sources of economic growth can be among others: focus on export-led policy, focus on high quality of goods, focus on science and research and foreign aid. Similar economic growth was evident towards the end of the 1980s which was replaced by a period of decline in the growth rate in consequence of German reunification in the period of transformation at the beginning of the 1990s. The last significant drop in GDP is evident from 2007 to 2009 in consequence of the economic crisis. The military time series indicates that we can notice differences in the development of German military expenditure, namely before 1991 and after 1991. Between the beginning of the first period characterized especially by building up the army and joining NATO, we can see fluctuating military expenditure from 2.8% to 5%. However, after 1991, it is possible to notice a downward trend in military expenditure from 2.2% to 1.4%. The armed forces restructuring for the purpose of peace-keeping, crises handling and fighting terrorism are being given priority. Germany has reduced its military expenditure by nearly 10% since 1995 and by nearly 6% since 2000.

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