1 ESTIMATION OF PARAMETERS

1.1 Point estimation

1.1.1 Exercises 1.1

1. Consider a randomly repeated laboratory observation of given constant μ which is based on a random sample $x_1, \ldots, x_n$, $E(x_i) = \mu$, $D(x_i) = \sigma^2$, $i = 1, \ldots, n$. Assume statistics
$$M_n = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{a} \quad L_n = \frac{x_1 + x_n}{2}.$$
   a) Verify, if M_n and L_n are unbiased estimators of the constant μ.
   b) Which estimator is better?
   c) Verify, if M_n and L_n are asymptotic unbiased estimators of the constant μ.
   d) Verify, if L_n is a consistent estimator of the constant μ.

2. Consider a random sample of 20 tank garrisons from a regiment. For each garrison was clocked the time needed to entrance into the tank (s): 10.5 10.8 11.2 10.9 10.4 10.6 10.9 11.0 10.3 10.8 10.6 11.3 10.5 10.7 10.8 10.9 10.9 10.8 10.7 11.0.
   a) Find the point estimate of the mean and the variance of the time which is needed to entrance into the tank for the whole monitored regiment.
   b) Determine the standard error of the estimate.

3. A new machine is monitored with the view to investigate the time needed to accomplish a work stage. The output is written in following table. Determine the point estimate of the mean and the point estimate of the standard deviation of the duration.

time (s)	30	31	32	33	34	35	36	37
number	1	7	33	63	38	5	1	2

4. A random sample of $n = 200$ observations selected from normal distribution is characterized by the sample mean $\overline{x} = 7.03$ and $\sum_{i=1}^{200}(x_i - \overline{x})^2 = 796$. Determine the sample standard deviation and the point estimation of the mean and the variance.

Solution.
1. a) yes; b) for $n \geq 3$ is M_n better; c) yes; d) no;
2. a) 10.780; 0.064 b) 0.056;
3. 33.060; 1.063;
4. 2; 7.03; 3.98.

1.2 Interval estimation

1.3 Interval estimation of normal population mean

1.3.1 Exercises 1.3

1. Consider a normal population and derive a left-sized confidence interval for the mean μ. Direction: use quantile $t_{1-\alpha}(n-1)$.
2. Consider a normal population and derive a left-sized confidence interval for the variance σ^2. Direction: use quantile $\chi^2_{1-\alpha}(n-1)$.
3. What effect will the change of the confidence have on the width of a confidence interval for a given size of a sample?
4. What effect will the change of the size of a sample have on the width of a confidence interval for a given confidence?
5. Determine a two-sized confidence interval for a mean, if you have sample size $n = = 10$, 20 or 30, sample mean $\overline{x} = 6.180$ and sample standard deviation $s = 0.399$. What effect will it have on the width of the confidence interval?
6. Consider we are interested in the maximum speed of a given type of a plane. A sample of 15 observations taken from this population produced the following data (m/s): 42.2 41.8 42.5 42.0 42.5 42.3 43.1 42.8 43.8 43.4 41.1 41.7 41.3 44.1 42.3. Assume the maximum speed has a normal distribution. Find
   a) the point estimate of the mean and the standard deviation of the maximum speed,
   b) the standard error of the estimate of the mean; interpret,
   c) a 95% two-sized confidence interval for the mean.
7. Consider we are interested in losses of corn and the loss is usually measured as the weight of not harvested seeds per 1 m^2. A random sample of 12 observations taken from this population produced the following data (g): 8.2 11.1 13.0 11.5 10.5 10.5 8.3 11.2 13.7 10.6 12.8 10.6. Assume the loss has a normal distribution. Find
   a) the estimate of average loss per 1 m^2 and determine the accuracy of this estimate,
   b) a value which is not exceeded by the average loss per 1 m^2 with the probability 95%,
   c) a point estimate of the variance, a point estimate of the standard deviation and a 95% two-sized confidence interval for the standard deviation,
   d) the estimate of the probability that the loss for one given m^2 is not greater than 13 g.

8. From the set of 1000 products ware randomly chosen 140 items in order to check distance measurement. The output is written in following table (mm):

length	37	38	39	40	41	42	43	44
number	3	13	21	43	27	18	11	4

   Assume the length has a normal distribution. Determine:

   a) the estimate of average length in the whole set and determine the accuracy of this estimate,
   b) an interval which includes average length of all products in the set with the probability 99% and determine the accuracy of this estimate,
   c) the accuracy of products by the means of a point estimate and a 99% confidence interval for the variance,
   d) the value of accuracy (mm) which is not exceeded with the probability 0.95 and interpret practical importance.

9. From a daily output of a baker ware randomly chosen 45 twists in order to check weight measurement, the required weight is 80 g for each twist. For the population, the sample mean is 80.332 and the sample standard deviation is 1.718. Assume the weight of twists is a random variable which has a normal distribution. Find:

   a) an average weight of twists in the whole output and determine the accuracy of this estimate,
   b) the variance of twists in the whole output (be careful about units),
   c) an average weight of twists by the help of a confidence interval, assume confidence is 95 %; is it correct to regard the deviation of the sample mean from the required weight 80 g as a random effect or is there a suspicion of a systematic deviation?

10. A sample of 100 randomly chosen steel bars was carried out in order to check a yield strength of given bars. Sample characteristics for the population are $\overline{x} = 286.4\ \mathrm{Nmm}^{-2}$ and $s^2 = 119.79\ \mathrm{N}^2\mathrm{mm}^{-4}$. Find a point estimate and an interval estimate of parameters μ and σ (confidence 0.99) providing that the monitored random variable has a normal distribution.

Solution.

5. a) 5.894–6.466; b) 5.993–6.367; c) 6.031–6.329;
6. a) 42.46; 0.863; b) 0.223; c) 41.982–42.938;
7. a) 11; 0.483; b) 11.867; c) 2.798; 1.673; 1.185–2.840; d) 0.885;
8. a) 40.4; 0.132; b) 40.060–40.740; 0.340; c) 2.443;1.839–3.432; d) 1.738;
9. a) 80.332; 0.256; b) 2.952; c) 79.816–80.848, random effect;
10. $\widehat{\mu} = 286.4\ \mathrm{Nmm}^{-2}$; $\widehat{\sigma} = 10.945\ \mathrm{Nmm}^{-2}$; 283.525–289.275; 9.237–13.353.

1.4 Interval estimation of population mean, large samples

1.4.1 Exercises 1.4

1. Derive a left-sized confidence interval for the mean μ. Direction: use quantile $u_{1-\alpha}$.
2. Solve the exercise 1.3.1/9 c) without the assumption that the random variable has a normal distribution. Compare outputs, comment.
3. Solve the exercise 1.3.1/10 only for parameter μ without the assumption that the random variable has a normal distribution. Compare outputs, comment.
4. A sample of 100 randomly chosen steel bars ware carried out in order to check the yield strength of given bars. Sample characteristics for the population are $\overline{x} = 250.2$ a $s = 11.74$ MPa. Providing the confidence is 99 %, determine
   a) an admissible error of the parameter μ,
   b) the number of observations so that the admissible error is not greater than 2.8?
5. Consider $x_1, \ldots, x_n$ is a random sample which is taken from a distribution $N(\mu; 0.04)$. What is the minimum number of observation so that the width of a confidence interval for μ is not greater than 0.16? Solve the exercise separately for
   a) the significance level $\alpha = 0.05$,
   b) the significance level $\alpha = 0.01$.

Solution.
2. 79.830–80.834, random effect;
3. $\widehat{\mu} = 286.4$ Nmm^{-2}; 283.581–289.219;
4. a) 3.024; b) 117;
5. a) 25; b) 42.

1.5 Interval estimation of population proportion

1.5.1 Exercises 1.5

1. Derive a left-sized confidence interval for the parameter π. Direction: use quantile $u_{1-\alpha}$.
2. A sample of 320 randomly chosen tin cans ware carried out in order to check the guarantee period of given tin cans. The outcome is that 59 tin cans are lapsed. Find the point estimate of the corresponding population proportion and construct a 95% confidence interval for the population proportion of lapsed tin cans. Determine a point estimate and an interval estimate of number N of lapsed tin cans for 20 000 stored tin cans.

3. Assume a tyre maker which produces, on average, 10 % unsatisfactory tyres. Construct a 95% confidence interval for the population proportion of unsatisfactory tyres, separately for the sample size
   a) 100,
   b) 400,
   c) 1600.

Solution.
2. 18.4 %; 14.2–22.6 %; $N = 3680$; $N \in (2840, 4520)$;
3. a) 0.041–0.159; b) 0.071–0.129; c) 0.085–0.115.