Basic Probability Theory

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Basic Probability Theory

- The random experiment and the random event
- The probability of the random experiment, the axioms of probability
- Definition of the probability (statistical, classical, geometrical)
- The conditional probability and independence
- The addition and multiplication law
- The law of total probability and Bayes' formula

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The Random Experiment and the Random Event

- **The random experiment** is every activity which results is not unambiguously determined by ambient conditions (e.g. rolling a die (tossing a coin), measurement of length, 100 metres run, lottery, ...).
- The result of the random experiment we call **outcome** (e.g. in two tosses of a coin we get 2 heads, the length is 25.7 cm, time is 13.8 s, the winning lottery ticket is B265430, ...).

An experiment is the process of observing a phenomenon that has variation in its outcomes.

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The Random Experiment and the Random Event

The **sample space** Ω associated with an experiment is the collection of all possible distinct outcomes of the experiment.
Each outcome is called an **elementary outcome,** a **simple event** or an **element of the sample space** – denote $\omega_1, \omega_2 \dots$.
An **event** is the set of elementary outcomes possessing a designated feature.

The sample space can

- discrete finite – roll a die and observe the number: $\Omega = \{1, 2, 3, 4, 5, 6\}$
- discrete infinite – roll a die repeatedly and count the number of rolls it takes until the first 6 appears $\Omega = \{1, 2, \dots\}$
- continuous – turn on a light bulb and measure its lifetime

$$\Omega = \langle 0, \infty)$$

The Random Experiment and the Random Event

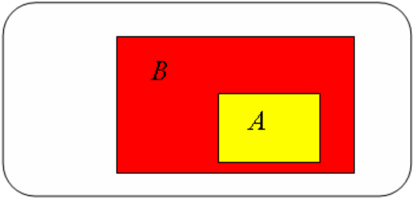
A subset of Ω, $A \subseteq \Omega$, is called an **event**.

If we roll a die and observe the number, two possible events are that we get odd outcome and that we get at least 4.

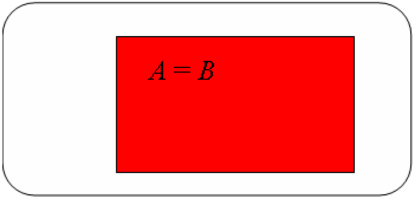
$$A = \{1, 3, 5\} \quad B = \{4, 5, 6\}$$

Basic set operations and:

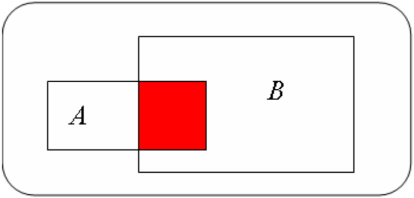
- an event A is a part of an event B ($A \subset B$)
  if A occurs then B occurs



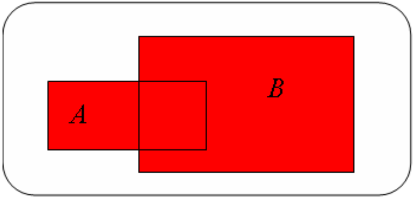
- events A and B are equivalent $(A = B)$
  A occurs if and only if B occurs



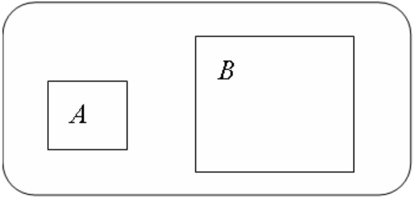
- the intersection of A and B ($A \cap B$)
  both A and B occur



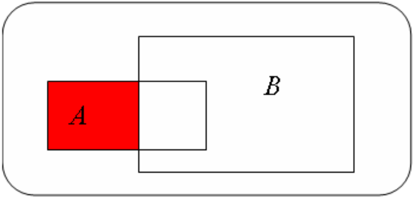
- the union of A and B ($A \cup B$)
  A or B (or both) occurs



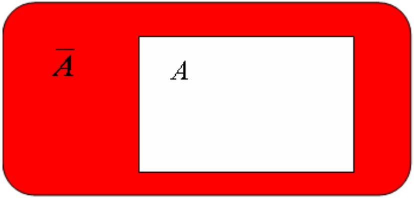
- A and B disjoint or mutually exclusive if they cannot both occur ($A \cap B = \emptyset$), where $\emptyset$ is an impossible event



- difference between$A$ and B $(A - B)$
  A occurs but not B

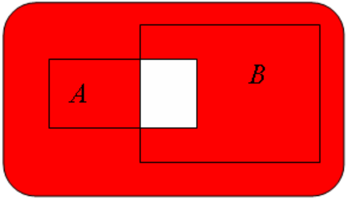


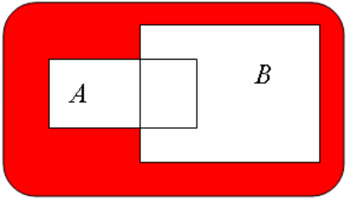
- the complement of A (denote $\overline{A}$)
  A does not occur



$$A \cup \overline{A} = \Omega, A \cap \overline{A} = \emptyset.$$

It easy to see that





$$\overline{A \cap B} = \overline{A} \cup \overline{B} \qquad \overline{A \cup B} = \overline{A} \cap \overline{B}$$

For more than two events we can write

$$\overline{\bigcap_i A_i} = \bigcup_i \overline{A_i} \quad \text{and} \quad \overline{\bigcup_i A_i} = \bigcap_i \overline{A_i}.$$

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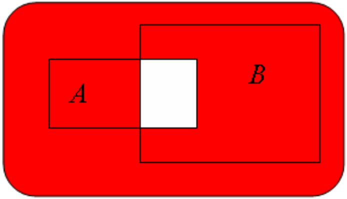

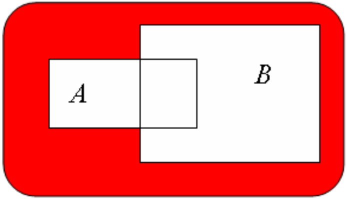

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The Axioms of Probability

- The probability of an event A is a non-negative number $(P(A) \geq 0)$.
- The probability of Ω is equal to 1 $(P(\Omega) = 1)$.
- If $A_1, A_2, \ldots$ is a sequence of pairwise disjoint events, that is, if $i \neq j$ then $A_i \cap A_j = \emptyset$, then

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$

$$P\left(\bigcup_{k=1}^{\infty} A_k\right) = \sum_{k=1}^{\infty} P(A_k)$$

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We can derive for the axioms another properties of probability:

- $0 \leq P(A) \leq 1$
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  (Ω and $\emptyset$ are disjoint events and $\Omega \cup \emptyset = \Omega$, we can write $P(\Omega) + P(\emptyset) = 1$, $1 + P(\emptyset) = 1 \Rightarrow P(\emptyset) = 0$.)

- $P(\overline{A}) = 1 - P(A)$

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Statistical Definition of Probability

Definition

Statistical definition of probability is given by the formula

$$P(A) \approx \frac{n(A)}{n},$$

where n is a number of observations, $n(A)$ denotes a frequency of an event A.

The ratio $\frac{n(A)}{n}$ we call a relative frequency. If the number of observation is big, the relative frequency of the event A is close to the probability of A.

Classical Definition of Probability

Definition

If the sample space $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ is finite and all elementary events have the same probability $P(\omega_i = \frac{1}{n})$ for $i = 1, 2, \dots, n)$, then the probability of an event A is

$$P(A) = \frac{m}{n},$$

where m is a number of favorable outcomes of A, where n is a number of possible outcomes of A.

Example 1

We have 32 cards (8 red, 8 black, 8 green and 8 blue), 4 cards are randomly chosen. What is the probability that we get 3 red cards?

Number of possible outcomes is

$$n = \binom{32}{4},$$

Number of favorable outcomes is

$$m = \binom{8}{3}\binom{24}{1},$$

$$P(A) = \frac{m}{n} = \frac{\binom{8}{3}\binom{24}{1}}{\binom{32}{4}} = 0.037.$$

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We roll 2 dice at once. What is the probability that the sum of numbers is equal to 3?

There are only 2 possibilities: 1+2 or 2+1.
We get the probability

$$P(A) = \frac{m}{n} = \frac{2}{36} = \frac{1}{18} = 0.056.$$

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It is known that among 100 products are 5 waste products. We choose randomly 3 products (not returning). What is the probability that we get 1 waste product?

Number of possible outcomes is

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Geometrical Definition of Probability

If sample space Ω is continuous it is not possible to calculate probability of an event A by the formula of classical probability (number of possible outcomes is infinite)

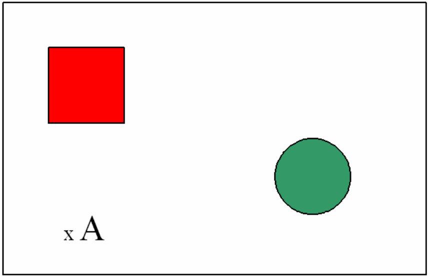
Definition

The sample space Ω is continuous creating some area which is bounded and closed and it's size is $V(\Omega)$ (described by the length, eventually the area or the volume). If an event $A \subset \Omega$ creates area with the size $V(A)$ then the probability of A is given by the formula

$$P(A) = \frac{V(A)}{V(\Omega)}.$$

Example 1

Let us have a rectangle $10 \times 5\,\text{cm}$. There are a square ($2 \times 2\,\text{cm}$), a circle (with diameter $2\,\text{cm}$) and a point A inside the rectangle. What is the probability that the randomly chosen point inside the rectangle is inside the square, inside the circle and is the point A?



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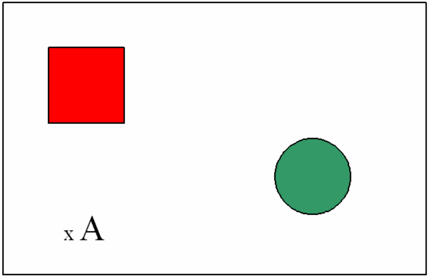

Example 1

The probability that the randomly chosen point is inside the square is:

- $V(A) = 2 \cdot 2 = 4\,\text{cm}^2$
- $V(\Omega) = 10 \cdot 5 = 50\,\text{cm}^2$
- $$P(A) = \frac{V(A)}{V(\Omega)} = \frac{4}{50} = 0.08$$

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Example 2

Let us assume that x, y are real numbers in the interval $[0; 1]$. What is the probability that their sum is smaller than 1 and their product is not bigger than $2/9$ at the same time?

- the sum smaller than 1

$$x + y < 1 \Leftrightarrow y < 1 - x$$

- the product smaller than $2/9$

$$xy < \frac{2}{9} \Leftrightarrow y < \frac{2}{9x}$$

Example 2

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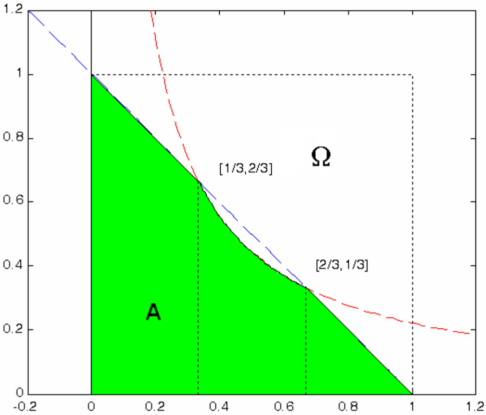

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